Examples: Solving Quadratic Equations of All Types

Any equation of the form $ax^2 + bx + c = 0$ *, with* $a \neq 0$ *, is called a quadratic equation.*

You should be able to look at an equation and determine the values of a, b and c.

Methods you may use: *Zero Product Principle (Factoring when equation = 0)*

Extracting Square Roots (For any quad. eq. with $b = 0$*)*

Quadratic Formula (For any quadratic equation!)

Completing the Square (For any quadratic equation with $a = 1$*)*

Zero Product Property *NOTE: To use this method, one side of the equation must equal 0.*

Notice in Examples 1 & 3 how the first step is to rewrite the equation so that one side is equal to 0.

1. $x^2 - 3x = 10$

   $x^2 - 3x - 10 = 0$

   $(x + 2)(x - 5) = 0$

   $x + 2 = 0$ or $x - 5 = 0$

   $x = -2$ or $x = 5$

2. $9x^2 - 6x = 0$

   $3x(3x - 2) = 0$

   $3x = 0$ *or* $3x - 2 = 0$

   $x = 0$ *or* $3x = 2$

   $x = 0$ *or* $x = \frac{3}{2}$

3. $x^3 = 16x$

   $x^3 - 16x = 0$

   $x(x^2 - 16) = 0$

   $x(x - 4)(x + 4) = 0$

   $x = 0$ or $x - 4 = 0$ or $x + 4 = 0$

   $x = 0$ or $x = 4$ or $x = -4$

Extracting Square Roots *NOTE: Use this method only when* $b = 0$*. After you get to the point where* $a = 1$*, take the square root of both sides. The answer is always* $\pm$*.*

4. $x^2 = 36$

   $\sqrt{x^2} = \pm\sqrt{36}$

   $x = \pm 6$

5. $3a^2 = 60$

   $a^2 = 20$

   $\sqrt{a^2} = \pm\sqrt{20}$

   $a = \pm 2\sqrt{5}$

6. $30 - 8a^2 = 28$

   $30 - 28 = 8a^2$

   $2 = 8a^2$

   $\frac{2}{8} = a^2$

$$\pm\sqrt{\frac{1}{4}} = \sqrt{a^2}$$

$$\pm\frac{1}{2} = a$$

Quadratic Formula

<u>If done properly</u>, this method is guaranteed to work for any quadratic equation. Of course, you must know the formula AND be able to use it correctly.

7. $2x^2 - 6x - 1 = 0$

$a = 2,\ b = -6,\ c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{6 \pm \sqrt{(-6)^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{6 \pm \sqrt{36 + 8}}{4}$$

$$x = \frac{6 \pm \sqrt{44}}{4}$$

$$x = \frac{6 \pm 2\sqrt{11}}{4}$$

$$x = \frac{3 \pm \sqrt{11}}{2}$$

Completing the Square

This method is used only when $a = 1$. You must be able to complete a perfect square trinomial as shown below.

8. $x^2 - 6x - 4 = 0$

$x^2 - 6x \quad = 4$

$x^2 - 6x + 9 = 4 + 9$

$(x - 3)^2 = 13$

$\sqrt{(x - 3)^2} = \pm\sqrt{13}$

$x - 3 = \pm\sqrt{13}$

$x = 3 \pm \sqrt{13}$