Simplify Complex Fractions

Def: A **complex fraction** is a fraction that has a fraction in its numerator and/or denominator.

Methods of Simplifying Complex Fractions:

1. Rewrite numerator and denominator as single fractions, then divide - in other words, multiply numerator by reciprocal of denominator.

2. Multiply both numerator and denominator by least common denominator (LCD) of every fraction in the numerator and denominator.

EXAMPLE: Simplify $\dfrac{\frac{3}{x}}{\frac{2}{x}+\frac{1}{x+4}}$.

Method 1: Start - rewrite denominator as a single fraction. Add the two fractions.

$$\frac{2}{x}+\frac{1}{x+4}=\frac{3x+8}{x(x+4)}$$

$$\frac{\frac{3}{x}}{\frac{2}{x}+\frac{1}{x+4}}=\frac{\frac{3}{x}}{\frac{3x+8}{x(x+4)}}$$ Continue with mult. by reciprocal of denom.

$$\frac{\frac{3}{x}}{\frac{3x+8}{x(x+4)}}=\frac{3}{x}\cdot\frac{x(x+4)}{3x+8}=\frac{3(x+4)}{3x+8} \text{ or } \frac{3x+12}{3x+8}$$

Method 2: Start - Find the LCD of the three fractions in the original expression.

LCD = $x(x+4)$

Multiply both num. and denom. by the LCD as shown below.

$$\frac{\frac{3}{x}}{\frac{2}{x}+\frac{1}{x+4}}\cdot\frac{x(x+4)}{x(x+4)}$$ OR write like this: $$\frac{\frac{3}{x}}{\frac{2}{x}+\frac{1}{x+4}}\cdot\frac{\frac{x(x+4)}{1}}{\frac{x(x+4)}{1}}$$

Simplify: $$\frac{\frac{3}{\cancel{x}}}{\frac{2}{x}+\frac{1}{x+4}}\cdot\frac{\frac{\cancel{x}(x+4)}{1}}{\frac{x(x+4)}{1}} = \frac{\frac{3(x+4)}{1}}{\frac{2(x+4)}{1}+\frac{x}{1}} = \frac{3(x+4)}{2x+8+x} = \frac{3(x+4)}{3x+8} \text{ or } \frac{3x+12}{3x+8}$$

Problems: Simplify each complex fraction.

1. $$\frac{\frac{3x}{4}-\frac{x}{6}}{\frac{5}{3}+\frac{5}{12}}$$

2. $$\frac{\frac{7}{a}+\frac{2}{3a}}{\frac{10}{a^2}+\frac{5}{2a}}$$

3. $$\frac{12}{\frac{4}{x}+\frac{1}{x-1}}$$

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1. $$\frac{\frac{3x}{4}-\frac{x}{6}}{\frac{5}{3}+\frac{5}{12}}=\frac{\frac{3x}{4}-\frac{x}{6}}{\frac{5}{3}+\frac{5}{12}}\cdot\frac{\frac{12}{1}}{\frac{12}{1}}=\frac{9x-2x}{20+5}=\frac{7x}{25}$$

2. $$\frac{\frac{7}{a}+\frac{2}{3a}}{\frac{10}{a^2}+\frac{5}{2a}}\cdot\frac{\frac{6a^2}{1}}{\frac{6a^2}{1}}=\frac{42a+4a}{60+15a}=\frac{46a}{15(a+4)}$$

3. $$\frac{12}{\frac{4}{x}+\frac{1}{x-1}}=\frac{12}{\frac{5x-1}{x(x-1)}}=\frac{12}{1}\cdot\frac{x(x-1)}{5x-1}=\frac{12x(x-1)}{5x-1}$$